Anchor-Agitated Systems: Power Input Correlation for Pseudoplastic and Thixotropic Fluids in Equilibrium

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Metzner and Otto (1957) assumed a linear relation between the average shear rate and rotational speed of an impeller, k_s being the proportionality constant. However, later investigations disclosed that k_s depends on the flow behavior index n. The purpose of this note is to report a new form of the function $k_s = k_s(n)$ for anchor agitators, as, according to our belief, validity of the existing relations was not supported for a sufficiently wide range of the flow behavior index. It has been proved experimentally (e.g., Houška, 1981) that power input prediction based on unreliable values of k_s may lead to errors as large as 50–100%.

THEORY

Making use of Metzner and Otto's (1957) basic assumption that

$$\dot{\gamma}_a = k_s N \tag{1}$$

Rieger and Novák (1973) derived an expression for the general parameter $C(n)$ in the well-known relation

$$Po = C(n)/Re_n \tag{2}$$

which holds in the viscous flow regime. Their expression, which is particularly well suited in determining k_s values from experimental data, is

$$C(n) = A\,k_s^{n-1} \tag{3}$$

Equation 3 offers a very simple means of testing the validity of the Metzner-Otto method. If experimental data obtained in a particular impeller-vessel configuration do not obey Eq. 3, the k_s parameter is very likely a function of n as well as of geometry. It is worth noting that the determination of k_s values from Eq. 3 yields more reliable results than from the formally equivalent relation $k_s = [C(n)/A]^{1/(n-1)}$, where experimental errors inherent in the determination of n are introduced into the exponent.

For thixotropic fluids, a similar procedure was developed in which a set of equilibrium (steady-state) experimental data consisting of equilibrium torque values and of equilibrium flow curves is employed. An equilibrium state of the mixing process is supposed to exist after a sufficiently long time of mixing at a constant rotational speed in the vessel or at a constant shear rate value in a rotational rheometer. In order to determine the rheological parameters, equilibrium rheometric data for a particular thixotropic fluid can always, at least within a limited range of shear rates, be approximated by the power law model,

$$\tau = K \cdot \dot{\gamma}^n \tag{4}$$

A combination of the equilibrium rheological parameters with $C(n)$ data obtained on a single impeller-vessel arrangement forms a complete set of values necessary to determine k_s values from Eq. 3 in spite of the fact that it has been derived for time-independent fluids.

If experimental data do not follow Eq. 3, a suitable form of the $k_s = k_s(n)$ function has to be found. Beckner and Smith (1966) suggested the simplest linear relation:

$$k_s = a \cdot (1 - n) \tag{5}$$

Rieger and Novák (1973) expressed $C(n)$ as a power function of the flow behavior index n

$$C(n) = C_1 \cdot n^{C_2} \tag{6}$$

which, after substitution into Eq. 3 with $C_1 = C(n = 1) \equiv A$, serves to determine the k_s. If neither of the aforementioned methods produces a satisfactory fit to the experimental data, the course of the $C(n)$ function may often be reproduced assuming the liquid motion in a given impeller-vessel combination to be similar to the rotational motion of a cylinder in the same liquid. Using this analogy, an expression of the general form

$$C(n) = p_1(p_2/n)^n \tag{7}$$

was adopted, which, after insertion into Eq. 3, again yields $k_s(n)$.

EXPERIMENTAL RESULTS

Geometry of the anchor impellers used in the experiments is shown in Figure 1. Details of the experimental setup are given elsewhere (Houška, 1981). Rheological parameters and

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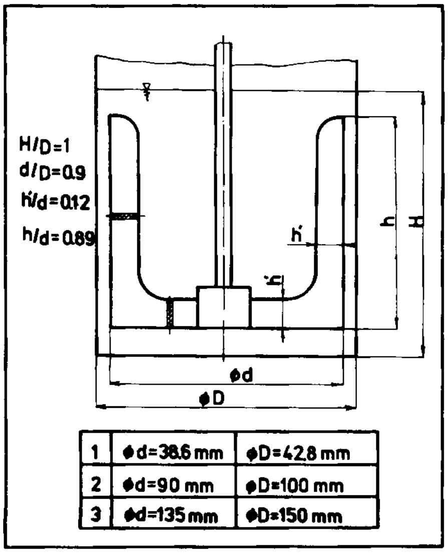

Figure 1. Dimensions of the anchor impellers (one pair of side arms).

average experimental values of $C(n)$ for all liquids are summarized in Table 1.

A thorough inspection of the results in Table 1 could lead to the conclusion that rheological data for some fluids are in contradiction, since different values of K and n are attributed to one and the same fluid. The reason for this discrepancy is that practically none of the pseudoplastic fluids used in the experiments followed the power law model closely enough. Therefore, in order to match the shear rates maintained in the mixing experiments, actual flow curves were approximated by the power law model parameters only locally. However, since the k_s values were not known in advance, mixing experiments had to be evaluated in the form of log M_k vs. log N plots wherefrom the first estimates of the n values were obtained. Only thereafter, were power law model parameters evaluated from the flow curves so as to achieve best agreement between n values obtained from the log M_k-log N plot and from the flow curve.

Finally, the power law model parameters were substituted into the definition of $C(n)$ (See Notation). According to our knowledge, this was the only way to obtain correct and objective values of $C(n)$ listed in Table 1. Experimental results with thixotropic fluids were evaluated in a similar way. In this case however, log M_k vs. log N plots were obtained from equilibrium

TABLE 1. SUMMARY OF EXPERIMENTAL RESULTS

Fluids	K, Pa · s^n	n	ρ, kg · m^{-3}	T, °C	Av. $C(n)$	No. of Experiments	d, mm
Newtonian:							
Silicone oil	0.259	1.0	840	19.9	213.7 ± 0.7	4	38.6
Mineral oil	0.9256	1.0	900	20.2	210.4 ± 1.0	14	38.6
Sugar syrup	2.612	1.0	1362	24.0	194.7 ± 2.5	14	90
	2.531	1.0	1362	24.3	191.8 ± 2.9	12	135
Pseudoplastic:							
Kaolin in water (50% w/w)	129.98	0.158	1424	24–32	16.92 ± 0.4	12	135
	129.14	0.0782	1424	24–32	15.06 ± 0.22	8	38.6
	130.32	0.1215	1424	24–32	16.46 ± 0.22	13	38.6
	130.15	0.1288	1424	24–32	17.52 ± 0.30	12	90
Polyox WSR in water (1% w/w)	0.794	0.62	1002	20.1	62.8 ± 2.7	4	38.6
Wallpaper gum (CMC* in water)	4.973	0.74	1015	20	99.72 ± 1.53	10	38.6
	7.64	0.61	1015	20	64.73 ± 1.37	5	38.6
PAA** in water (10% w/w)	67.50	0.816	1110	20	96.83 ± 2.14	11	38.6
	80.04	0.53	1110	20	43.42 ± 1.14	9	38.6
CMC in water (3.8% w/w)	1.2	0.874	1014	20.1	133.99 ± 2.15	13	90
	1.305	0.810	1014	20.1	124.70 ± 1.81	6	90
PAA in water (6% w/w)	15.05	0.744	1016	20.1	80.96 ± 1.68	10	90
	12.2	0.9595	1016	20.1	193.3 ± 1.96	7	90
	13.53	0.854	1016	20.1	120.75 ± 1.53	10	135
CMC in water (8% w/w)	12.25	0.66	1024	20.2	62.76 ± 1.03	6	90
	11.64	0.70	1024	20.2	73.53 ± 1.77	7	90
	11.637	0.732	1024	20.2	86.49 ± 1.30	12	135
CMC in water (3.6% w/w)	1.07	0.905	1013	20.1	136.82 ± 4.60	19	135
Thixotropic (equilibrium values):							
Neoponite No. 1	46.7	0.484	1929	21	44.0 ± 1.1	11	38.6
Neoponite No. 2	35.9	0.497	1927	21	39.2 ± 1.1	10	90
					39.9 ± 3.5	10	135
					23.5 ± 1.0	11	38.6
Wallpaper paint	12.6	0.249	1439	21	19.9 ± 0.7	11	90
					21.7 ± 0.6	9	135
Laponite in water (5% w/w)					8.5 ± 0.5	9	38.6
	300	0.022	1021	21	5.4 ± 0.5	9	90
					5.8 ± 0.4	7	135
					Total	305	

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*Carboxymethyl cellulose.

**Polyacrylamide.

torque values obtained after sufficiently long shearing times. From the relation log $C(n)$ vs. $(1 - n)$ the parameters $A = 200 \pm 2$ and $k_s = 22.8 \pm 2.4$ were obtained, using linear regression with a correlation coefficient −0,983.

Since it is clear from Figure 2 that accuracy of reproducing the data with Eq. 3 decreases for low values of n, it appeared to be necessary to seek another method to express $k_s = k_s(n)$. Due to the fact that Eq. 6 proved not to be valid in the whole pseudoplastic range of n, Eq. 7 was chosen as a basis for further evaluation. Using data from Table 1, empirical constants p_1 and p_2 were determined, keeping in mind that $p_1 p_2 = A$. For the particular anchor agitator, the following expression was found using nonlinear regression:

$$C(n) = 5.74\,(35/n)^n \tag{8}$$

Substituting Eq. 8 into 3, with $C(1) = A$ yields

$$k_s = 35 \cdot n^{n/(1-n)} \tag{9}$$

DISCUSSION

Equation 9 may easily be compared with predictions of other authors who investigated a similar geometry of the anchor agitator. Substituting into the Beckner and Smith (1966) correlation $d/D = 0.9$, we have $k_s = 31(1 - n)$. Rieger and Novák's (1973) prediction, for the same geometry, gives $k_s = n^{2.21/(n-1)}$.

From Figure 3 it is clear that, for $n > 0.5$, Eq. 9 compares well with the results of Rieger and Novák. For decreasing values of n, their k_s predictions approach infinity, which contradicts our as well as Beckner and Smith's results. On the other hand, Beckner and Smith's prediction of zero k_s values for n approaching unity seems to be equally unrealistic. Therefore it may be concluded that the form of Eq. 9 yields k_s values in the whole pseudoplastic range for anchor agitators with a higher degree of reliability than the previous correlations.

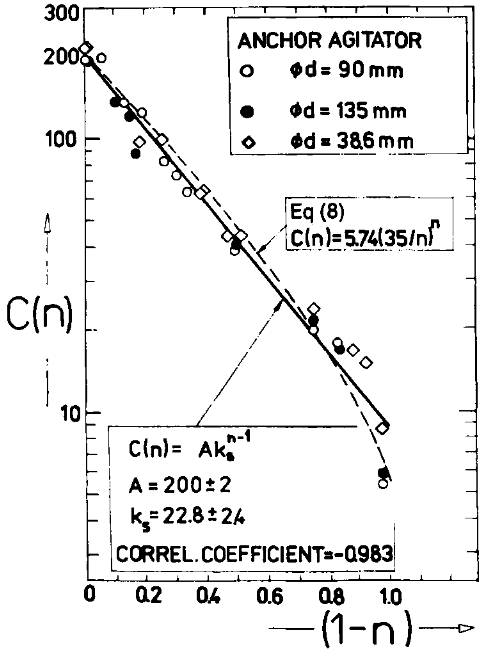

Figure 2. Reproduction of $C(n)$ by Eq. 3.

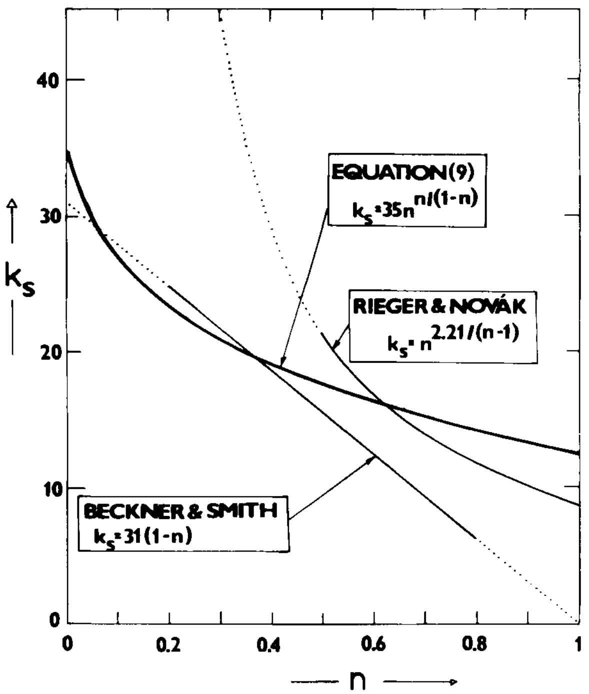

Figure 3. Dependency of k_s on the flow behavior index; comparison with other authors.

AN IMPROVED METHOD FOR PREDICTING THE POWER INPUT

Power input prediction for a particular anchor agitator based on Eq. 9 is believed to be more accurate than that which relies upon the mean value of k_s only. This statement is supported by experimental evidence that shows that use of the latter method may lead to an under- or overestimated size of the driving unit, depending on whether the flow behavior index of the agitated pseudoplastic fluid is large or small.

Therefore, in order to respect the generally obvious relation $k_s = k_s(n)$ for fluids that do not strictly obey the power law model, the following design procedure is recommended:

a. A suitable choice of a rotational speed N is made.

b. First estimate of k_s is obtained, e.g., from Eq. 9, for a mean value of n taken from the flow curve.

c. $\dot\gamma_a$ is calculated from Eq. 1.

d. Drawing a tangent to the flow curve at $\dot\gamma_a$, a more accurate value of $n = d\log\tau/(d\log\dot\gamma)$ is obtained.

e. Substitution of n into Eq. 9 yields a new k_s. This procedure is repeated starting from step c until convergence is reached.

f. The value of $\dot\gamma_a$ resulting from the iteration procedure yields the apparent viscosity $\mu_a = K\dot\gamma_a^{n-1}$, which, after substitution into the modified Reynolds under Re', locates the power number Po on the corresponding Newtonian Po vs. Re' characteristic.

THIXOTROPIC FLUIDS

The procedure outlined above also proved successful in predicting equilibrium values of the power input in agitating thixotropic fluids. This is of practical importance for low viscosity thixotropes exhibiting short times of the structural breakdown, where equilibrium is reached rapidly and startup effects are relatively unimportant. For highly viscous thixotropes with long times of structural breakdown, the opposite is true. For these fluids, the startup period usually cannot be neglected and the whole time course of the power input decay is required for a safe design of the motor drive (Edwards et al., 1976; Šesták et al., 1982). In any event, regardless of the rate of structural destruction, determination of reliable k_s values is essential at the beginning of any procedure leading to the power input value.

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NOTATION

A = constant reflecting geometry in the Newtonian power characteristics for the creeping flow regime $Po = A/Re$
a = empirical constant
C_1, C_2 = empirical constants
$C(n)$ = dimensionless function of geometry and n, $P/(K N^{n+1} d^3)$
D = vessel diameter, m
d = agitator diameter, m
H = liquid height in the vessel, m
h = agitator height, m
h' = arm width, m
K = fluid consistency, $\text{Pa} \cdot \text{s}^n$
k_s = Metzner-Otto parameter
M_k = torque, $\text{N} \cdot \text{m}$
N = rotational speed, s^{-1}
n = flow behavior index
P = power input, W
Po = power number, $P/(\rho \cdot N^3 \cdot d^5)$
p_1, p_2 = empirical constants
Re = Reynolds number for Newtonian fluids, $(\rho \cdot N \cdot d^2)/\mu$
Re' = modified Reynolds number, $(\rho \cdot N \cdot d^2)/\mu_a$
Re_n = Reynolds number for power law fluids, $(N^{2-n} \cdot d^2 \cdot \rho)/K$
T = temperature, °C

Greek Letters

$\dot{\gamma}_a$ = average shear rate, s^{-1}
$\dot{\gamma}$ = shear rate, s^{-1}
τ = shear stress, Pa
μ_a = apparent viscosity, $\text{Pa} \cdot \text{s}$
ρ = fluid density, $\text{kg} \cdot \text{m}^{-3}$

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